Convergence of discounted solutions of the Hamilton-Jacobi equation

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This is a joint work with Andrea Davini, Renato Iturriaga, and Maxime Zavidovique.

We consider a Hamiltonian $H : \mathbf{R}^n \times \mathbf{R}^n \to \mathbf{R}, (x,p) \mapsto H(x,p)$ that is $\mathbf{Z}^n$ periodic in the first variable x, and convex superlinear in the second variable p.

For $\lambda > 0$ we consider (viscosity) solutions of the discounted Hamilton-Jacobi equation

$$\lambda u_\lambda + H(x, Du_\lambda(x)) = c[0],$$

where $c[0]$ is the unique constant c such that the stationary Hamilton-Jacobi equation

$$H(x, Du(x)) = c$$

has a viscosity solution. It s well-known that u_λ is unique and that u_λ accumulates on viscosity solutions of the stationary Hamilton-Jacobi equation, when $\lambda \to 0$.

We address the problem of actual convergence of u_λ when $\lambda \to 0$.

Using weak KAM theory, we can show that u_λ converges to a unique viscosity solution of the stationary Hamilton-Jacobi equation.

A few years ago, a partial solution to this problem was obtained by the speaker and Renato Iturriaga. Andrea Davini and Maxime Zavidovique recently completed the program.

We will recall the elements from Aubry-Mather and weak KAM theory that are necessary to understand the lecture.