SEMILINEAR ELLIPTIC PROBLEMS AND LINEAR SCHRÖDINGER OPERATORS IN LIPSCHITZ DOMAINS.

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We consider equations of the form $(*) \quad -\Delta u + g(x,u) = 0$ in a bounded Lipschitz domain Ω in $\mathbb{R}^N$. Basic assumptions on g: (i)$g \in C(\Omega \times \mathbb{R})$, (ii) $g(x,\cdot)$ is monotone increasing and convex and $g(x,0) = 0$ for every $x \in \Omega$; (iii) There exists a constant $a > 0$ such that, for every positive solution u of $(*)$, $g(x,u(x)) \leq au(x)\text{dist}(x,\partial\Omega)^{-2}$. If u is a positive solution of $(*)$ then u satisfies the Schrödinger equation $(**) \quad -\Delta u + Vu = 0$ where $V = g(x,u)/u$. Note that by (ii) $V(x) \leq a\,\text{dist}(x,\partial\Omega)^{-2}$. The connection between these two equations enables us to apply tools of harmonic analysis and potential theory to the study of boundary value problems for $(*)$.

Our results include a necessary and sufficient condition on a positive Radon measure μ on $\partial\Omega$ for the existence and uniqueness of a solution of the Dirichlet b.v.p. for $(*)$ with boundary data μ.

Our conditions on g hold for a wide class of functions including power and exponential nonlinearities.